



## HARFORD COUNTY PUBLIC SCHOOLS GRADE 4 MATHEMATICS CURRICULUM

[CLICK HERE](#) for the Maryland College and Career Ready Standards for Grade 4 Mathematics.

### Topic 1: Generalize Place Value Understanding

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

#### Enduring Understandings

- **Number Uses, Classification, and Representation** – Numbers can be used for different purposes, and numbers can be classified and represented in different ways.
- **Numbers** – The set of real numbers is infinite and ordered. Whole numbers, integers, and fractions are real numbers. Each real number can be associated with a unique point on the number line.
- **Base-Ten Numeration System** - The base-ten numeration system is a scheme for recording numbers using the digits 0-9, groups of 10, and place value. In this system, the place value to the immediate left of a given place is 10 times as great as the place value of the given place. For example, 1,000 is ten times as great as 100, and 100 is ten times as great as 10. These attributes of our numeration system can be used to compare and round numbers.
- **Equivalence** – Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
- **Comparison and Relationships** - Numbers, expressions, measures, and objects can be compared and related to other numbers, expressions, measures, and objects in different ways.
- **Estimation** – Numbers can be approximated by replacing numbers with other numbers that are close and easy to compute mentally.

#### Essential Questions

- How are greater numbers written?
- How can whole numbers be compared?
- How are place values related?

Lesson Title	Lesson Overview	Standards
Numbers Through One Million	Our number system is based on groups of ten. Whenever we get 10 in one place value, we move to the next greater place value.	4.NBT.A.2
Place Value Relationships	In a multi-digit whole number, a digit in one place represents ten times what it would represent in the place immediately to its right.	4.NBT.A.1
Compare Whole Numbers	Place value can be used to compare numbers.	4.NBT.A.2



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Round Whole Numbers	Rounding whole numbers is a process for finding the multiple of 10, 100 and so on closest to a given number.	4.NBT.A.3
Problem Solving: Construct Arguments	Good math thinkers use math to explain why they are right. They can talk about the math that others do, too.	MP.3



## HARFORD COUNTY PUBLIC SCHOOLS GRADE 4 MATHEMATICS CURRICULUM

### Topic 2: Fluently Add and Subtract Multi-Digit Whole Numbers

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

#### Enduring Understandings

- **Numbers** – The set of real numbers is infinite and ordered. Whole numbers, integers, and fractions are real numbers. Each real number can be associated with a unique point on the number line.
- **Base-Ten Numeration System** - The base-ten numeration system is a scheme for recording numbers using the digits 0-9, groups of 10, and place value.
- **Equivalence** – Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
- **Operations Meanings and Relationships** - There are multiple interpretations of addition, subtraction, multiplications, and division of rational numbers, and each operation is related to other operations.
- **Estimation**- Numbers can be approximated by replacing numbers with other numbers that are close and easy to compute mentally. Estimating sums and differences is a useful technique to quickly solve mathematical problems and understand the value of numbers used in real-world situations. There is more than one way to estimate sums and differences. Students have been estimating with whole numbers since grade 3.
- **Properties** - For a given set of numbers there are relationships that are always true, called properties, and these are the rules that govern arithmetic and algebra.
- **Algorithms** - There are algorithms for performing each of the operations with rational numbers. Strategies and algorithms use equivalence, place value, and properties of operations to transform calculations into simpler ones.
- **Variables, Expressions, and Equations** – Letters and symbols, called variables, can be used to stand for a number or any number from a particular set of numbers. Some mathematical and real-world situations can be represented using variables, operations, and numbers in expressions and equations.
- **Solving Equations** – Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

#### Essential Questions

- How can sums and differences of whole numbers be estimated? • What are standard procedures for adding and subtracting whole numbers?

Lesson Title	Lesson Overview	Standards
Finding Sums and Differences with Mental Math	Representing numbers and numerical expressions in equivalent forms can make some calculations easy to do mentally. There is more than one way to do a mental calculation.	4.NBT.B.4



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Estimate Sums and Differences	There is more than one way to estimate a sum or difference. Estimation is helpful for checking to see if an answer is reasonable or to find an approximate answer when an exact answer is not necessary.	4.NBT.B.4 4.OA.A.3
Add Whole Numbers	The standard addition algorithm for adding 3-digit numbers is an extension to the standard algorithm for adding 2-digit numbers.	4.NBT.B.4 4.OA.A.3
Add Greater Numbers	The standard addition algorithm for multi-digit numbers breaks the calculation into simpler calculations using place value.	4.NBT.B.4 4.OA.A.3
Subtract Whole Numbers	The standard subtraction algorithm for multi-digit numbers is an efficient strategy that can be used to subtract any two numbers. The calculations are done by place value starting with the ones, then the tens, and so on, regrouping as needed.	4.NBT.B.4 4.OA.A.3
Subtract Greater Numbers	The standard algorithm for subtraction breaks the calculation into simpler calculations using place value starting with the ones, then the tens, and so on.	4.NBT.B.4 4.OA.A.3
Subtract Across Zeros	The standard algorithm for subtraction breaks the calculation into simpler calculations using place value starting with the ones, then the tens, and so on.	4.NBT.B.4 4.OA.A.3
Problem Solving: Reasoning	Good math thinkers know how to think about words and numbers to solve problems.	MP.2



## HARFORD COUNTY PUBLIC SCHOOLS GRADE 4 MATHEMATICS CURRICULUM

### Topic 3: Use Strategies and Properties to Multiply By 1-Digit Numbers

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

#### Enduring Understandings

- **Numbers** – The set of real numbers is infinite and ordered. Whole numbers, integers, and fractions are real numbers. Each real number can be associated with a unique point on the number line.
- **Base-Ten Numeration System** - The base-ten numeration system is a scheme for recording numbers using the digits 0-9, groups of 10, and place value.
- **Equivalence** – Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
- **Operations Meanings and Relationships** - There are multiple interpretations of addition, subtraction, multiplications, and division of rational numbers, and each operation is related to other operations.
- **Estimation** - Numbers can be approximated by replacing numbers with other numbers that are close and easy to compute mentally. Estimating products is a useful technique to quickly solve mathematical problems and understand the value of numbers used in real-world situations. There is more than one way to estimate a product. Students have been estimating with whole numbers since grade 3.
- **Properties** - For a given set of numbers there are relationships that are always true, called properties, and these are the rules that govern arithmetic and algebra.
- **Algorithms** - There are algorithms for performing each of the operations with rational numbers. Strategies and algorithms use equivalence, place value, and properties of operations to transform calculations into simpler ones. Instruction extends students' understanding of the area model and distributive property with single digit factors learned in grade 3 to multiplying a multi-digit factor and a single digit factor in this topic.
- **Variables, Expressions, and Equations** – Letters and symbols, called variables, can be used to stand for a number or any number from a particular set of numbers. Some mathematical and real-world situations can be represented using variables, operations, and numbers in expressions and equations.
- **Solving Equations** – Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.
- **Patterns, Relations, and Functions** – Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

#### Essential Questions

- How can you multiply numbers by multiples of 10, 100, and 1,000?
- How can you multiply whole numbers?



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Lesson Title	Lesson Overview	Standards
Multiply by Multiples of 10, 100, and 1,000	Basic facts and place-value patterns can be used to find products when one factor is 10, 100, or 1,000.	4.NBT.B.5
Estimate Products	Rounding is one way to estimate products.	4.OA.A.2 4.OA.A.3
Use Arrays and Partial Products to Multiply	The expanded algorithm for multiplication can be represented with arrays. In the algorithm, numbers are broken apart using place value, and the parts are used to find partial products.	4.NBT.B.5
Use Area Models and Partial Products to Multiply	Area models and properties of multiplication can be used to simplify computation.	4.NBT.B.5
More Use Area Models and Partial Products to Multiply	The expanded algorithm for multiplication breaks numbers apart using place value, and the parts are used to find partial products. The partial products are then added together to find the product.	4.NBT.B.5 4.OA.A.3
Mental Math Strategies for Multiplication	Properties of multiplication and place-value understanding can be used to multiply without paper and pencil.	4.NBT.B.5
Choose a Strategy to Multiply	Students can use the Distributive Property, area models, and other methods to find a product.	4.NBT.B.5
Problem Solving: Model with Math	Good math thinkers choose and apply math they know to show and solve problems from everyday life.	MP.4



## HARFORD COUNTY PUBLIC SCHOOLS GRADE 4 MATHEMATICS CURRICULUM

### Topic 4: Use Strategies and Properties to Multiply by 2-Digit Numbers

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

#### Enduring Understandings

- **Base-Ten Numeration System** - The base-ten numeration system is a scheme for recording numbers using the digits 0-9, groups of 10, and place value.
- **Equivalence** – Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
- **Operations Meanings and Relationships** - There are multiple interpretations of addition, subtraction, multiplications, and division of rational numbers, and each operation is related to other operations.
- **Estimation** - Numbers can be approximated by replacing numbers with other numbers that are close and easy to compute mentally. Estimating products is a useful technique to quickly solve mathematical problems and understand the value of numbers used in real-world situations. There is more than one way to estimate a product. Students have been estimating with whole numbers since grade 3.
- **Properties** - For a given set of numbers there are relationships that are always true, called properties, and these are the rules that govern arithmetic and algebra.
- **Algorithms** - There are algorithms for performing each of the operations with rational numbers. Strategies and algorithms use equivalence, place value, and properties of operations to transform calculations into simpler ones. Instruction extends students' understanding of the area model and distributive property with multiplication of a multi-digit factor and a single-digit factor to multiplication with two two-digit factors.
- **Variables, Expressions, and Equations** – Letters and symbols, called variables, can be used to stand for a number or any number from a particular set of numbers. Some mathematical and real-world situations can be represented using variables, operations, and numbers in expressions and equations.
- **Solving Equations** – Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.
- **Patterns, Relations, and Functions** – Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

#### Essential Questions

- How can you use a model to multiply?
- How can you use the Distributive Property to multiply?
- How can you use multiplication to solve problems?



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Lesson Title	Lesson Overview	Standards
Multiply Multiples of 10	Basic facts and place-value patterns can be used to mentally multiply a 2-digit number by a multiple of 10.	4.NBT.B.5
Use Models to Multiply 2-Digit Numbers by Multiples of 10	Place-value blocks, area models, and arrays provide ways to visualize and find products.	4.NBT.B.5
Estimate: Use Rounding or Compatible Numbers	Products of 2-digit by 2-digit multiplication problems can be estimated by replacing each factor with the closest multiple of ten.	4.NBT.B.5 4.OA.A.3
Arrays and Partial Products	The expanded algorithm for multiplying with 2-digit numbers is an extension of the expanded algorithm for multiplying with 1-digit numbers.	4.NBT.B.5 4.OA.A.3
Area Models and Partial Products	The Distributive Property can be used to multiply two 2-digit numbers by breaking the computation down into 4 simpler products and adding the partial products together.	4.NBT.B.5
Use Partial Products to Multiply By 2-Digit Numbers	The expanded algorithm for multiplication can be represented with arrays. In the algorithm, numbers are broken apart using place value, and the parts are used to find partial products.	4.NBT.B.5 4.OA.A.3
Problem Solving: Make Sense and Persevere	Good math thinkers make sense of problems and think of ways to solve them. If they get stuck, they don't give up.	MP.1



## HARFORD COUNTY PUBLIC SCHOOLS GRADE 4 MATHEMATICS CURRICULUM

### Topic 5: Use Strategies and Properties to Divide By 1-Digit Numbers

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

#### Enduring Understandings

- **Base-Ten Numeration System** - The base-ten numeration system is a scheme for recording numbers using the digits 0-9, groups of 10, and place value.
- **Equivalence** – Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
- **Operations Meanings and Relationships** - There are multiple interpretations of addition, subtraction, multiplications, and division of rational numbers, and each operation is related to other operations.
- **Estimation** - Numbers can be approximated by replacing numbers with other numbers that are close and easy to compute mentally. Estimating quotients is a useful technique to quickly solve mathematical problems and understand the value of numbers used in real-world situations. Using compatible numbers is one of many strategies that can be used. Estimating quotients using compatible numbers is new to students in grade 4.
- **Properties** - For a given set of numbers there are relationships that are always true, called properties, and these are the rules that govern arithmetic and algebra.
- **Algorithms** - There are algorithms for performing each of the operations with rational numbers. Strategies and algorithms use equivalence, place value, and properties of operations to transform calculations into simpler ones. Sharing items, area models and arrays, and partial quotients are some ways to represent division with multi-digit whole numbers.
- **Variables, Expressions, and Equations** – Letters and symbols, called variables, can be used to stand for a number or any number from a particular set of numbers. Some mathematical and real-world situations can be represented using variables, operations, and numbers in expressions and equations.
- **Solving Equations** – Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.
- **Patterns, Relations, and Functions** – Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

#### Essential Questions

- How can mental math be used to divide? • How can quotients be estimated? • How can the steps for dividing be explained?



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Lesson Title	Lesson Overview	Standards
Mental Math: Find Quotients	Basic facts and place-value patterns can be used to divide multiples of 10 and 100 by 1-digit numbers.	4.NBT.B.6
Mental Math: Estimate Quotients	There is more than one way to estimate a quotient. Substituting compatible numbers is an efficient technique for estimating quotients.	4.NBT.B.6 4.OA.A.3 4.NBT.B.5
Mental Math: Estimate Quotients for Greater Dividends	There is more than one way to estimate a quotient. Using place-value patterns and compatible numbers are efficient techniques for estimating quotients.	4.NBT.B.6 4.OA.A.3 4.NBT.B.5
Interpret Remainders	When dividing, the remainder must be less than the divisor. When solving a real-world problem, the kind of questions asked determines how to interpret the remainder.	4.NBT.B.6 4.OA.A.3
Use Partial Quotients to Divide	Division with partial quotients involves breaking apart the dividend, dividing the parts, and adding the partial quotients.	4.NBT.B.6
Use Partial Quotients to Divide: Greater Dividends	Division with partial quotients involves breaking apart the dividend, dividing the parts, and adding the partial quotients.	4.NBT.B.6
Use Sharing to Divide	Sharing is one way to think about division.	4.NBT.B.6
Continue Sharing to Divide	You can use estimation and place value to divide.	4.NBT.B.6 4.OA.A.3
Choosing a Strategy to Divide	There are many ways to perform division, including mental math, models, partial quotients, and sharing.	4.NBT.B.6
Problem Solving: Model with Math	Good math thinkers choose and apply math they know to show and solve problems from everyday life.	MP.4



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**Topic 6: Use Operations with Whole Numbers to Solve Problems**

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

**Enduring Understandings**

- **Comparison and Relationships** - Numbers, expressions, measures, and objects can be compared and related to other numbers, expressions, measures, and objects in different ways.
- **Operations Meanings and Relationships** - There are multiple interpretations of addition, subtraction, multiplication, and division of rational numbers, and each interpretation is related to other operations. For example, multiplication as comparison is the focus of some lessons in topic 6. The word problems in lessons include other interpretations of multiplication as well, such as equal groups.
- **Basic Facts and Algorithms** – There is more than one algorithm for each of the operations with rational numbers. Some strategies for basic facts and most algorithms for operations with rational numbers, both mental math and paper and pencil, use equivalence to transform calculations into simpler ones.
- **Variables, Expressions, and Equations** – Letters and symbols, called variables, can be used to stand for a number or any number from a particular set of numbers. Some mathematical and real-world situations can be represented using variables, operations, and numbers in expressions and equations.
- **Solving Equations** – Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

**Essential Questions**

- How is comparing with multiplication different from comparing with addition?
- How can you use equations to solve multi-step problems?

Lesson Title	Lesson Overview	Standards
Solve Comparison Problems	Both addition and multiplication can be used to make comparisons. Bar diagrams and equations can be used to show both situations and to distinguish between them.	4.OA.A.1 4.OA.A.2 4.NBT.B.5
Continue to Solve Comparison Problems	Bar diagrams and equations can be used to solve problems involving multiplicative comparison.	4.OA.A.1 4.OA.A.2 4.NBT.B.5 4.NBT.B.6



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Model Multi-Step Problems	Bar diagrams and equations can be used to model and solve multi-step problems.	4.OA.A.3 4.OA.A.2 4.NBT.B.4 4.NBT.B.5 4.NBT.B.6
More Model Multi-Step Problems	Multi-step problems can be modeled and solved in more than one way.	4.OA.A.3 4.OA.A.2 4.NBT.B.4 4.NBT.B.5 4.NBT.B.6
Solve Multi-Step Problems	Equations can represent problems, and are helpful in answering both hidden questions and the original question in a problem.	4.OA.A.3 4.OA.A.2 4.NBT.B.4 4.NBT.B.5 4.NBT.B.6
Problem Solving: Make Sense and Persevere	Good math thinkers make sense of problems and think of ways to solve them. If they get stuck, they don't give up.	MP.1



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**Topic 7: Factors and Multiples**

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

**Enduring Understandings**

- **Number Uses, Classification, and Representation** - Numbers can be classified and represented in different ways. Students classify a number as a factor of another number, a multiple of another number, as a prime number, or as a composite number.
- **Equivalence** – Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
- **Properties** - For a given set of numbers there are relationships that are always true, called properties, and these are the rules that govern arithmetic and algebra.
- **Patterns, Relations, and Functions** – Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.

**Essential Questions**

- How can you use arrays or multiplication to find the factors of a number?
- How can you identify prime and composite numbers?
- How can you find multiples of a number?

Lesson Title	Lesson Overview	Standards
Understand Factors	Factors of a number $n$ can be shown by arranging $n$ counters into rows with the same number of counters in each row. The number of rows and counters in each row are factors if $n$ .	4.OA.B.4 4.NBT.B.5
Factors	Factors of a number can be found in pairs by thinking about multiplication.	4.OA.B.4 4.NBT.B.5
Problem Solving: Repeated Reasoning	Good math thinkers look for things that repeat, and they make generalizations.	MP.8
Prime and Composite Numbers	Prime numbers have exactly 2 factors and composite numbers have more than 2.	4.OA.B.4 4.NBT.B.5
Multiples	The product of any nonzero whole number and a given nonzero is a multiple of both. Factors and multiples are closely related.	4.OA.B.4 4.NBT.B.5



## HARFORD COUNTY PUBLIC SCHOOLS GRADE 4 MATHEMATICS CURRICULUM

### Topic 8: Extend Understanding of Fraction Equivalence and Ordering

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

#### Enduring Understandings

- **Number Uses, Classification, and Representation** - Numbers can be used for different purposes, and numbers can be classified and represented in different ways.
- **Numbers** – The set of real numbers is infinite and ordered. Whole numbers, integers, and fractions are real numbers. Each real number can be associated with a unique point on the number line.
- **Equivalence** - Any number can be represented in an infinite number of ways that have the same value. This topic focuses on using models as well as mathematical procedures to recognize and generate equivalent fractions.
- **Comparison** - Numbers, expressions, measures, and objects can be compared and related to other numbers, expressions, measures, and objects in different ways. Using benchmark fractions and reasoning about the size of fractions builds students' number sense. Finding equivalent fractions with the same numerator or same denominator can be helpful when comparing fractions but is not always necessary. Comparisons are valid only when the fractions refer to the same whole.
- **Estimation** – Numbers can be approximated by numbers that are close.

#### Essential Questions

- What are some ways to name the same part of a whole?
- How can you compare fractions with unlike numerators and denominators?

Lesson Title	Lesson Overview	Standards
Equivalent Fractions: Area Models	Two fractions that represent the same part of the same whole are equivalent. The two fractions are different names for the same number.	4.NF.A.1
Equivalent Fractions: Number Lines	The same fractional amount can be represented by an infinite set of different but equivalent fractions.	4.NF.A.1
Generate Equivalent Fractions: Multiplication	When the numerator and denominator of a fraction are multiplied by the same whole number greater than 1, it is the same as multiplying the fraction by 1. This gives an equivalent fraction because multiplying by 1 does not change the value of a number.	4.NF.A.1 4.NBT.B.5



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Generate Equivalent Fractions: Division	When the numerator and denominator of a fraction are divided by a common factor, the result is an equivalent fraction.	4.NF.A.1 4.OA.B.4 4.NBT.B.6
Use Benchmarks to Compare Fractions	One way to compare two fractions that are part of the same whole is by comparing each to a benchmark fraction such as $\frac{1}{2}$ .	4.NF.A.2
Compare Fractions	When two fractions have the same denominator, the fraction with the greater numerator is greater. When two fractions have the same numerator, the fraction with the lesser denominator is greater.	4.NF.A.1 4.NF.A.2 4.NBT.B.5
Problem Solving: Construct Arguments	Good math thinkers use math to explain why they are right. They can talk about the math that others do, too.	MP.3



## HARFORD COUNTY PUBLIC SCHOOLS GRADE 4 MATHEMATICS CURRICULUM

### Topic 9: Understand Addition and Subtraction of Fractions

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

#### Enduring Understandings

- **Number Uses, Classification, and Representation** - Numbers can be used for different purposes, and numbers can be classified and represented in different ways.
- **Numbers** – The set of real numbers is infinite and ordered. Whole numbers, integers, and fractions are real numbers. Each real number can be associated with a unique point on the number line.
- **Equivalence** - Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
- **Operation Meanings and Relationships** – There are multiple interpretations of addition, subtraction, multiplication, and division of rational numbers, and each operation is related to other operations. The meanings of addition and subtraction are the same for fractions and whole numbers, even though algorithms for calculating sums and differences can be different.

Addition:

- add to
- put together

Subtraction:

- take from
- take apart
- compare / difference

- **Estimation** – Numbers can be approximated by numbers that are close. Numerical calculations can be approximated by replacing numbers with other numbers that are close and easy to compute with mentally. Estimation using benchmark fractions and number sense is a valuable technique that helps determine the reasonableness of sums and differences.
- **Properties** – For a given set of numbers there are relationships that are always true, called properties, and these are the rules that govern arithmetic and algebra.
- **Algorithms** – There is more than one algorithm for each of the operations with rational numbers. Complex calculations involving fractions and mixed numbers can be broken into simpler equivalent calculations.
- **Variables, Expressions, and Equations** – Letters and symbols, called variables, can be used to stand for a number or any number from a particular set of numbers. Some mathematical and real-world situations can be represented using variables, operations, and numbers in expressions and equations.
- **Solving Equations** – Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

#### Essential Questions

- How do you add and subtract fractions and mixed numbers with like denominators?
- How can fractions be added and subtracted on a number line?



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Lesson Title	Lesson Overview	Standards
Model Addition of Fractions	Tools can be used to show addition of fractions as joining parts of the same whole.	4.NF.B.3a 4.NF.B.3d
Decompose Fractions	A fraction $\frac{a}{b}$ , where $a > 1$ , can be decomposed into the sum of two or more unit or non-unit fractions in one or more ways where the sum of the fractions is equal to the original fraction.	4.NF.B.3b
Add Fractions with Like Denominators	Two fractions can be joined or added to find the total. There is a general method for adding fractions with like denominators.	4.NF.B.3a 4.NF.B.3d
Model Subtraction of Fractions	Tools can be used to show subtraction of fractions as separating a part from the same whole.	4.NF.B.3a 4.NF.B.3d
Subtract Fractions with Like Denominators	The difference between two fractions with like denominators can be found by separating one fractional amount from the other. There is a general method for subtracting fractions with like denominators.	4.NF.B.3a 4.NF.B.3d
Add & Subtract Fractions with Like Denominators	Fraction addition and subtraction can be thought about as joining and separating segments on a number line. They can also be thought about as counting forward or counting backward on the number line.	4.NF.B.3a 4.NF.B.3d
Model Addition and Subtraction of Mixed Numbers	Adding and subtracting mixed numbers is an extension of the ideas and procedures for adding and subtracting fractions.	4.NF.B.3c 4.NF.B.3d
Add Mixed Numbers	Two procedures for adding mixed numbers both involve changing the calculation to a simpler equivalent calculation.	4.NF.B.3c 4.NF.B.3d
Subtract Mixed Numbers	Two procedures for subtracting mixed numbers both involve changing the calculation to a simpler equivalent calculation. These are extensions of the same procedures used for adding mixed numbers with like denominators.	4.NF.B.3c 4.NF.B.3d
Problem Solving: Model with Math	Good math thinkers choose and apply math they know to show and solve problems from everyday life.	MP.4



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**Topic 10: Extend Multiplication Concepts to Fractions**

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

**Enduring Understandings**

- **Numbers** – The set of real numbers is infinite and ordered. Whole numbers, integers, and fractions are real numbers. Each real number can be associated with a unique point on the number line.
- **Equivalence** - Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
- **Operation Meanings and Relationships** – There are multiple interpretations of addition, subtraction, multiplication, and division of rational numbers, and each operation is related to other operations. The meaning of multiplication is the same for fractions and whole numbers, even though algorithms for calculating sums and differences can be different.
- **Algorithms** - Complex calculations involving fractions can be broken into simpler equivalent calculations.
- **Variables, Expressions, and Equations** – Letters and symbols, called variables, can be used to stand for a number or any number from a particular set of numbers. Some mathematical and real-world situations can be represented using variables, operations, and numbers in expressions and equations.
- **Solving Equations** – Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations so solutions can be found.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

**Essential Questions**

- How can you describe a fraction using a unit fraction?
- How can you multiply a fraction by a whole number?

Lesson Title	Lesson Overview	Standards
Fractions as Multiples of Unit Fractions	Any fraction $\frac{a}{b}$ can be written as $a$ times the unit fraction $\frac{1}{b}$ .	4.NF.B.4a
Multiply a Fraction by a Whole Number: Use Models	Models and equations can be used to represent problems and compute products of whole numbers and fractions.	4.NF.B.4b 4.NF.B.4a 4.NF.B.4c
Multiply a Fraction by a Whole Number: Use Symbols	Models and equations can be used to represent problems and compute products of whole numbers and fractions.	4.NF.B.4b 4.NF.B.4a 4.NF.B.4c



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Solve Time Problems	The standard algorithms for adding, subtracting, multiplying, and dividing can be used to solve time problems.	4.MD.A.2 4.NF.B.4c 4.NF.B.3d 4.MD.A.1
Problem Solving: Model with Math	Good math thinkers choose and apply math they know to show and solve problems from everyday life.	MP.4



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**Topic 11: Represent and Interpret Data on Line Plots**

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

**Enduring Understandings**

- **Numbers** – The set of real numbers is infinite and ordered. Whole numbers, integers, and fractions are real numbers. Each real number can be associated with a unique point on the number line.
- **Data Collection and Representation** - Some questions can be answered by collecting and analyzing data, and the question to be answered determines the data that need to be collected and how best to collect the data. Data can be represented visually using tables, charts, and graphs. The type of data determines the best choice of visual representation.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

**Essential Questions**

- How can you solve problems using data on a line plot?
- How can you make a line plot?

Lesson Title	Lesson Overview	Standards
Read line plots	A line plot organizes data on a number line and is useful for showing how data are distributed.	4.MD.B.4 4.NF.B.3d
Make line plots	A line plot organizes data on a number line and is useful for showing how data are distributed.	4.MD.B.4 4.NF.A.1 4.NF.A.2 4.NF.B.3d
Use line plots to solve problems	Data from line plots can be used to solve problems.	4.MD.B.4 4.NF.B.3d
Problem Solving: Critique Reasoning	Good math thinkers use math to explain why they are right. They can talk about the math that others do, too.	MP.3



## HARFORD COUNTY PUBLIC SCHOOLS GRADE 4 MATHEMATICS CURRICULUM

### Topic 12: Understand and Compare Decimals

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

#### Enduring Understandings

- **Number Uses, Classification, and Representation** – Numbers can be used for different purposes, and numbers can be classified and represented in different ways. Decimals are another way to represent amounts also named using fractions.
- **Numbers** – The set of real numbers is infinite and ordered. Whole numbers, integers, and fractions are real numbers. Each real number can be associated with a unique point on the number line.
- **Base-Ten Numeration System** - The base-ten numeration system is a scheme for recording numbers using the digits 0-9, groups of 10, and place value.
- **Equivalence** – Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
- **Comparison and Relationships** - Numbers, expressions, measures, and objects can be compared and related to other numbers, expressions, measures, and objects in different ways.
- **Operation Meanings and Relationships** – There are multiple interpretations of addition, subtraction, multiplication, and division of rational numbers, and each operation is related to other operations.
- **Algorithms** - There are algorithms for performing each of the operations with rational numbers. Strategies and algorithms use equivalence, place value, and properties of operations to transform calculations into simpler ones.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

#### Essential Questions

- How can you write a fraction as a decimal?
- How can you locate points on a number line?
- How do you compare decimals?

Lesson Title	Lesson Overview	Standards
Fractions and Decimals	A decimal is another way to represent a fraction.	4.NF.C.6
Fractions and Decimals on the Number Line	Points on a number line can represent fractions and decimals. A fraction and a decimal tell the distance a point is from 0 on the number line.	4.NF.C.6 4.MD.A.2
Compare Decimals	Place value can be used to compare decimals.	4.NF.C.7 4.MD.A.2



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Add Fractions with Denominators of 10 and 100	Fractions with denominators of 10 can be written as equivalent fractions with denominators of 100. Fractions with like denominators can be added.	4.NF.C.5
Solve Word Problems involving Money	Fractions and decimals can be used to represent amounts of money. Pictorial models and equations can represent problems involving money.	4.MD.A.2
Problem Solving: Look for and Use Structure	Good math thinkers look for relationships in math to help solve problems.	MP.7



## HARFORD COUNTY PUBLIC SCHOOLS GRADE 4 MATHEMATICS CURRICULUM

### Topic 13: Measurement: Find Equivalence in Units of Measure

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

#### Enduring Understandings

- **Equivalence** – Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value. In grade 4, students convert measurements from larger units to smaller units. They work with customary and metric units of length, capacity, and mass.
- **Comparison and Relationships** - Numbers, expressions, measures, and objects can be compared and related to other numbers, expressions, measures, and objects in different ways.
- **Variables, Expressions, and Equations** – Letters and symbols, called variables, can be used to stand for a number or any number from a particular set of numbers. Some mathematical and real-world situations can be represented using variables, operations, and numbers in expressions and equations.
- **Solving Equations** – Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.
- **Ratios and Proportionality** – When mathematical or real-world quantities have a relationship that can be stated as “for every  $x$  units of the first quantity there are  $y$  units of the second quantity,” this relationship can be described as a ratio. Proportionality involves a relationship in which the ratio of two quantities remains constant as the corresponding values of the quantities change. In a proportional relationship there are an infinite number of ratios equal to the lowest terms or constant ratio.
- **Measurement** – Some attributes of objects are measurable and can be quantified using unit amounts.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

#### Essential Question

- How can you convert from one unit to another?
- How can you be precise when solving math problems?

Lesson Title	Lesson Overview	Standards
Equivalence with Customary Units of Length	To convert from a larger unit of length to a smaller unit, multiply the number of larger units by the conversion factor, that is, the number of smaller units in each larger unit.	4.MD.A.1 4.MD.A.2 4.OA.A.3 4.NF.B.3d 4.NF.B.4c



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Equivalence with Customary Units of Capacity	To convert from a larger unit of capacity to a smaller unit, multiply the number of larger units by the conversion factor, that is, the number of smaller units in each larger unit.	4.MD.A.1 4.MD.A.2 4.OA.A.3 4.NF.B.3d 4.NF.B.4c
Equivalence with Customary Units of Weight	To convert from a larger unit of weight to a smaller unit, multiply the number of larger units by the conversion factor, that is, the number of smaller units in each larger unit.	4.MD.A.1 4.MD.A.2 4.OA.A.3 4.NF.B.3d 4.NF.B.4c
Equivalence with Metric Units of Length	To convert from a larger unit of length to a smaller unit, multiply the number of larger units by the conversion factor, that is, the number of smaller units in each larger unit.	4.MD.A.1 4.MD.A.2 4.OA.A.3 4.NF.C.7
Equivalence with Metric Units of Capacity and Mass	To convert from a larger unit of capacity or mass to a smaller unit, multiply the number of larger units by the conversion factor, that is, the number of smaller units in each larger unit.	4.MD.A.1 4.MD.A.2 4.OA.A.3
Solve Perimeter and Area Problems	Some problems can be solved by applying the formula for the perimeter of a rectangle or the formula for the area of a rectangle.	4.MD.A.3 4.OA.A.3 4.MD.A.2 4.NF.B.4c
Problem Solving: Precision	Good math thinkers are careful about what they write and say, so their ideas about math are clear.	MP.6



**HARFORD COUNTY PUBLIC SCHOOLS  
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**Topic 14: Algebra: Generate and Analyze Patterns**

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

**Enduring Understandings**

- **Ratios and Proportionality** – When mathematical or real-world quantities have a relationship that can be stated as “for every  $x$  units of the first quantity there are  $y$  units of the second quantity,” this relationship can be described as a ratio. Proportionality involves a relationship in which the ratio of two quantities remains constant as the corresponding values of the quantities change. In a proportional relationship there are an infinite number of ratios equal to the lowest terms or constant ratio.
- **Patterns, Relations, and Functions** - Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways. Given rules describing a relationship, students generate and describe the corresponding number or shape pattern.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

**Essential Questions**

- How can you use a rule to continue a pattern?
- How can you use a table to extend a pattern?
- How can you use a repeating pattern to predict a shape?

Lesson Title	Lesson Overview	Standards
Number Sequences	Rules can be used to create or extend number sequences that form a pattern. Those patterns sometimes have features not described by the rule.	4.OA.C.5 4.NBT.B.4 4.OA.B.4
Patterns: Number Rules	Rules can be used to create or extend patterns in tables. Patterns sometimes have features not described by the rule.	4.OA.C.5 4.OA.B.4 4.NBT.B.5 4.NBT.B.6
Patterns: Repeating Shapes	It is possible to predict a shape in a repeating pattern of shapes.	4.OA.C.5 4.OA.A.3 4.NBT.B.6
Problem Solving: Look for and Use Structure	Good math thinkers look for relationships in math to help solve problems.	MP.7



**HARFORD COUNTY PUBLIC SCHOOLS  
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**Topic 15: Geometric Measurement: Understand Concepts of Angles and Measurement**

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

**Enduring Understandings**

- **Variables, Expressions, and Equations** – Letters and symbols, called variables, can be used to stand for a number or any number from a particular set of numbers. Some mathematical and real-world situations can be represented using variables, operations, and numbers in expressions and equations.
- **Solving Equations** – Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.
- **Geometric Figures** - Two- and three-dimensional objects with or without curved surfaces can be described, classified, and analyzed by their attributes. An object's location in space can be described quantitatively.
- **Measurement** - Some attributes of objects are measurable and can be quantified using unit amounts.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

**Essential Questions**

- What are some common geometric terms? • How can you measure angles?

Lesson Title	Lesson Overview	Standards
Lines, Rays, and Angles	Line segments and rays are sets of points that describe parts of lines and angles. Angles are classified by their measure.	4.MD.C.5a 4.G.A.1
Understand Angles and Unit Angles	The measure of an angle depends upon the fraction of a circle that the angle turns through.	4.MD.C.5a 4.NF.A.1 4.NF.B.3b
Measure with Unit Angles	The unit for measuring angles in $1^\circ$ , the unit angle.	4.MD.C.5b 4.MD.C.5a
Measure and Draw Angles	The unit for measuring angles in $1^\circ$ , the unit angle. A protractor can be used to measure angles.	4.MD.C.6 4.MD.C.5b
Add and Subtract Angle Measures	Angle measures can be added and subtracted.	4.MD.C.7 4.NBT.B.4
Problem Solving: Use Appropriate Tools	Good math thinkers know how to pick the right tools to solve math problems.	MP.5



## HARFORD COUNTY PUBLIC SCHOOLS GRADE 4 MATHEMATICS CURRICULUM

### Topic 16: Lines, Angles & Shapes

**Primary Resource:** *enVision Mathematics Grade 4*, Savvas Learning Company, 2024.

#### Enduring Understandings

- **Geometric Figures** - Two- and three-dimensional objects with or without curved surfaces can be described, classified, and analyzed by their attributes. An object's location in space can be described quantitatively.
- **Practices, Processes, and Proficiencies** - Mathematics content and processes are applied to solve problems.

#### Essential Question

- How can you classify triangles and quadrilaterals?
- What is line symmetry?

Lesson Title	Lesson Overview	Standards
Lines	Lines can be classified as parallel, intersecting, or perpendicular.	4.G.A.1
Classify Triangles	Triangles are classified by their sides and by their angles.	4.G.A.2 4.OA.C.5 4.MD.C.5 4.G.A.1
Classify Quadrilaterals	Quadrilaterals are classified by their sides and by their angles.	4.G.A.2 4.G.A.1
Line Symmetry	A shape that can fold along a line into matching parts is line symmetric.	4.G.A.3
Draw Shapes with Line Symmetry	A shape that can fold along a line into matching parts is line symmetric.	4.G.A.3
Problem Solving: Critique Reasoning	Good math thinkers use math to explain why they are right. They can talk about the math that others do, too.	MP.3